I hope you have gone through the theory of probability from your book. I will not replicate that theory here but will assume that you already know it. Instead, what we will do now is take some tricky questions on probability and try and find out the various ways in which they can be solved. Hope they give you ideas and takeaways for other questions too!

Question: At the shooting range, the probability that Robert will hit the target in any one shot is 25%. If he takes four shots one after another, what is the probability that he will hit the target?

Solution: The first thing to understand here is the figure of 25%. What does it mean? It means that the probability of Robert hitting the target in a shot is 1/4. So he is expected to miss the target 3 out of 4 times i.e. his probability of not hitting the target in a shot is 3/4.

To hit the target, he needs to hit it at least once. When he takes four shots in succession, if he hits the target in any one shot, the target is hit. After that, whether he hits it or misses, it doesn't matter at all.

Another thing, the question asks you for the probability that he will hit the target in 4 shots? Is this probability 1 then? (since he is expected to hit it once in every four shots) No. Each shot he takes is independent of the shot he took before that and the shot he will take after that. In every shot the probability of hitting the target remains 1/4. Not that after he takes 3 shots and fails to hit the target, his probability of hitting the target in the fourth shot will become 1; it will still remain 1/4.

A similar case that might make this concept clearer is 'tossing a coin'. What is the probability of getting 'heads' in a toss? 1/2, I am sure you agree. So if I toss a coin once and get 'tails', does it mean I will definitely get 'heads' on the next toss? No, because the two tosses are independent of each other. My probability of getting 'heads' is still 1/2 on the second toss.

So we have established that the probability is not 1. Let's try and find out what the probability is. You can do it in various ways. Let's look at some of them.

Method 1: This is the simplest method and should be used. (The other two methods I will discuss are for intellectual purposes. In some questions, these other methods might come in handy.)

To hit the target, you need to hit it at least once in the 4 shots. You would not have hit the target, if you do not hit it in any of the 4 shots. Let's find the probability that he will not hit the target in any of the four shots. We can then subtract it from 1 to get the probability that he will hit the target in at least one shot.

Probability of not hitting the target in a shot = 3/4

Probability of not hitting the target in any of the 4 successive shots = (3/4)*(3/4)*(3/4)*(3/4) = 81/256

(We multiply the '(3/4)'s together because they are independent events.)

Probability of hitting the target at least once = 1 – (81/256) = 175/256

This is the required probability!

Method 2:

You hit the target if you hit it at least once. Say, if you hit the target on the first shot, you are done no matter what you do on the rest of the 3 shots. If you do not hit the target on the first shot, but hit it on the second shot, again, you are done and so on…

Probability of hitting the target on the first shot = 1/4

Probability of not hitting the target on the first shot but hitting it on the second shot = (3/4) * (1/4)

Probability of not hitting the target on the first and second shot but hitting it on the third shot = (3/4) * (3/4) * (1/4)

Probability of not hitting the target on the first, second and third shot but hitting it on the fourth shot = (3/4) * (3/4) * (3/4) * (1/4)
He could have hit the target in any one of the four ways given above.

Therefore, probability of hitting the target = (1/4) + (3/4)*(1/4) + (3/4)*(3/4)*(1/4) + (3/4)*(3/4)*(3/4)*(1/4) = (1*64 + 3*16 + 9*4 + 27)/256 = 175/256

As expected, the answer we get is the same as the one obtained in method 1 above.

Method 3: There is another way of thinking about this.

Say H = Hit and M = Miss

He can hit the target by hitting it only once. He can also hit the target by hitting it on two shots. He can also hit the target by hitting it on three shots and so on…

If he hits the target only once, it means he is successful (probability 1/4) once and unsuccessful (probability 3/4) three times. This can happen in various ways e.g. HMMM or MHMM or MMHM or MMMH. The hits and misses can be arranged in 4!/3! ways. (You arrange 4 things out of which 3 are identical in 4!/3! ways.)

Probability of hitting the target only once = (1/4)*(3/4)*(3/4)*(3/4) * 4!/3! = 27*4/256

Similarly, probability of hitting the target twice = (1/4)*(1/4)*(3/4)*(3/4) * 4!/(2!*2!) = 54/256
(Why do we multiply by 4!/(2!*2!)? Same logic as above. You arrange 4 things out of which 2 pairs are identical in 4!/(2!*2!) ways. HHMM or HMHM or MHHM etc.)

Using the same logic, probability of hitting the target three times = (1/4)*(1/4)*(1/4)*(3/4) * 4!/3! = 12/256

and probability of hitting the target four times = (1/4)*(1/4)*(1/4)*(1/4)*1 = 1/256

To get the probability of hitting the target, all we need to do now is add up all the four cases above.

Therefore, probability of hitting the target = 27*4/256 + 54/256 + 12/256 + 1/256 = 175/256

Of course, the answer still remains 175/256

I am sure different methods work for different people. Similarly, different methods work for different questions too. My suggestion is to be comfortable with all three methods. In weeks to come, I might point back to one of these 3 methods and show you why it works best for a particular question. Till then, keep practicing!